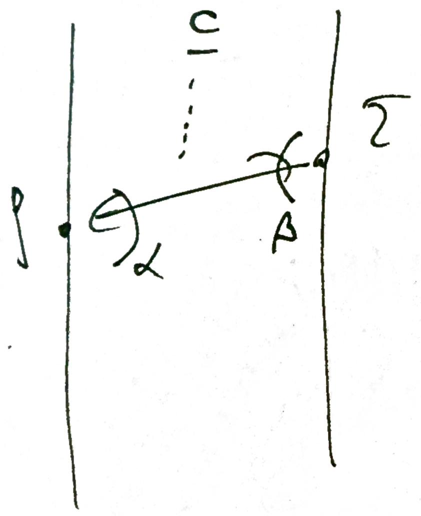
$a := \#$ of ρs $= \binom{n+1}{n^\varepsilon+1}\binom{n}{n^\varepsilon}(n-n^\varepsilon)!$

choices for dom(ρ) ... rng(ρ) maps between them

$b := \#$ of τs $= \binom{n+1}{n^\varepsilon+1+k/2}\binom{n}{n^\varepsilon+k/2}(n-n^\varepsilon+k/2)!$



$\alpha := \#$ of $\tau \supseteq \rho$ for a given ρ

$= \binom{n^\varepsilon+1}{k/2}\binom{n^\varepsilon}{k/2}(k/2)!$

$\beta := \#$ of $\rho \subseteq \tau$ for a given τ

$= \binom{n-n^\varepsilon+k/2}{k/2}$

CLEARLY: $\underline{a \cdot \alpha = b \cdot \beta}$, SO $\frac{a}{b} = \frac{\beta}{\alpha}$

$$\frac{\beta}{\alpha} = \frac{\dfrac{(n-n^{\varepsilon}+\ell/2)!}{(\ell/2)!\,(n-n^{\varepsilon})!}}{\dfrac{(n^{\varepsilon}+1)!}{(\ell/2)!\,(n^{\varepsilon}+1-\ell/2)!} \cdot \dfrac{(n^{\varepsilon})!}{(n^{\varepsilon}-\ell/2)!\,(\ell/2)!}} = \frac{(n-n^{\varepsilon}+\ell/2)!\,(n^{\varepsilon}+1-\ell/2)!\,(n^{\varepsilon}-\ell/2)!}{(n^{\varepsilon}+1)!\,(n^{\varepsilon})!\,(n-n^{\varepsilon})!} =$$

$$= \frac{(n-n^{\varepsilon}+\ell/2)\cdot\ldots\cdot(n-n^{\varepsilon}+1)}{(n^{\varepsilon}+1)\cdots(n^{\varepsilon}+1-\ell/2+1)\cdot n^{\varepsilon}\cdot\ldots\cdot(n^{\varepsilon}-\ell/2+1)} > \frac{(n-n^{\varepsilon})^{\ell/2}}{(n^{\varepsilon})^{\ell}}$$

$$\geq \left(n^{1/2-\varepsilon}\right)^{4}$$

SO WE WANT TO SHOW: $\boxed{\left(n^{1/2-\varepsilon}\right)^{4} > 5\cdot\left(\ell^{1/2}\cdot n^{\varepsilon}\right)^{4}}$

$\Leftarrow$

30.

i.e.

$$n^{1/2-\varepsilon} > 2 \cdot k^{1/2} \cdot n^{\varepsilon} \quad , \text{ as } \quad s \le 2^4$$

AS $k = n^{\delta}$ WE WANT:

$$n^{1/2-\varepsilon} > 2 \cdot n^{\delta/2} \cdot n^{\varepsilon}$$

WHICH FOLLOWS FOR $n \gg 1$ FROM:

$$1/2 - \varepsilon > \delta/2 + \varepsilon$$

$$\Updownarrow$$

$$1 > 4\varepsilon + \delta$$

AND THAT FOLLOWS FROM $0 < \delta < \varepsilon < 1/5$. $\square$