CVIČENÍ 2 MA 2, 6.5.2020

Budeme zkoumat konvergenci newtonových integrálů.

TYPICKÁ ÚLOHA

Nechť $a, b \in \mathbb{R}^*$, $a < b$, $f: (a,b) \to \mathbb{R}$ je spojitá. Konverguje $\int_a^b f(x)\,dx$?

ZÁKLADNÍ NÁSTROJE

- srovnávací kritérium
- limitní srovnávací kritérium
- věta o substituci

$\boxed{1}$ $\int_0^1 x^\alpha\,dx$ K $\iff$ $\alpha > -1$

$\boxed{2}$ $\int_1^\infty x^\alpha\,dx$ K $\iff$ $\alpha < -1$

$\boxed{3}$ $\int_0^{1/e} \frac{|\log x|^\alpha}{x} dx = \int_{-\infty}^{-1} |t|^\alpha dt$ K $\Leftrightarrow$ $\underline{\alpha < -1}$

$\log x = t$ $\qquad \int_1^{\infty} y^\alpha dy$

$\frac{1}{x} dx = dt$

$$\int_{-\infty}^{-1} (-t)^\alpha dt = -\int_{\infty}^{1} y^\alpha dy = \int_1^{\infty} y^\alpha dy$$

$-t = y$

$-dt = dy$

$\boxed{4}$ $\int_1^e \frac{(\log x)^\alpha}{x} dx$

$\log x = t$ $\qquad \int_0^1 t^\alpha dt$

$\boxed{5}$ $\displaystyle\int_1^\infty \underbrace{x^\alpha e^{\beta x}}_{f>0}\,dx \qquad \alpha, \beta \in \mathbb{R}$

$\underline{\beta < 0 \quad K}$

$\beta < \beta' < 0$

$g(x) = e^{\beta' x} \qquad \displaystyle\int_1^\infty e^{\beta' x}\,dx = \left[\frac{1}{\beta'} e^{\beta' x}\right]_1^\infty = 0 - \frac{1}{\beta'} e^{\beta'} \in \mathbb{R}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{x^\alpha e^{\beta x}}{e^{\beta' x}} = \lim_{x\to\infty} \frac{x^\alpha}{e^{(\beta'-\beta)x}} = 0$$

$\underline{\beta = 0}$: $\displaystyle\int_1^\infty x^\alpha\,dx \quad K \Leftrightarrow \alpha < -1$

$\underline{\beta > 0}$: $\displaystyle\int_1^\infty e^{\beta' x}\,dx = \infty$

$\beta' > 0$

$g(x) = e^{\beta' x} \qquad \beta > \beta' > 0 \qquad \displaystyle\lim_{x\to\infty} \frac{x^\alpha e^{\beta x}}{e^{\beta' x}} = \infty$

$\boxed{6}$ $\displaystyle\int_0^\infty \underbrace{x^{-3/4} e^{-\sqrt{x}}}_{f(x)} dx$

<u>bod 0:</u> $g(x) = x^{-3/4}$ $\quad -3/4 > -1$

$$\lim_{x \to 0+} \frac{f(x)}{g(x)} = \lim_{x \to 0+} e^{-\sqrt{x}} = 1 \in (0, \infty)$$

$$\int_0^1 f \ K \iff \int_0^1 g \ K$$

$$\int_0^1 g \ K \Rightarrow \int_0^1 f \ K$$

<u>bod ∞:</u> $g(x) = x^{-3/4} \cdot \frac{1}{x} = x^{-4/4}$ $\quad -4/4 < -1$

$\displaystyle\int_1^\infty g \ K$

$\Downarrow$

$\displaystyle\int_1^\infty f \ K$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{e^{-\sqrt{x}}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{x}{e^{\sqrt{x}}}$$

$$= \lim_{x \to \infty} \frac{(\sqrt{x})^2}{e^{\sqrt{x}}} = 0$$

$\frac{y}{e^y} \to 0, \ y \to \infty$