CVIČENÍ Z MATEMATICKÉ ANALÝZY 2, 27.5.2020

$\boxed{1}$ Pro diferenciální rovnici $y' = \frac{-(1+y^2)x}{1+x^2}$ nalezněte

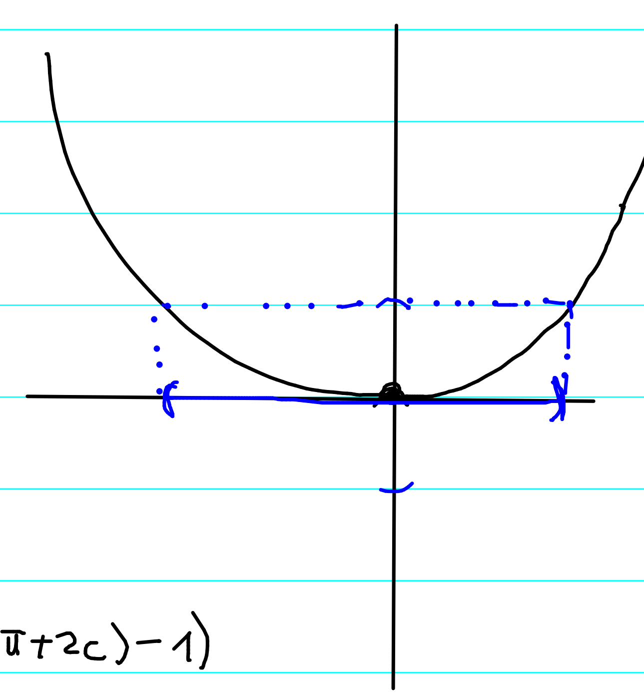
- všechna maximální řešení,
- maximální řešení procházející bodem $[0,1]$.

$h(x) = \frac{-x}{1+x^2}$ $\qquad g(y) = 1+y^2$

$\mathcal{D}(h) = \mathbb{R}$ $\qquad \mathcal{D}(g) = \mathbb{R}$ $\qquad g > 0$

neexistují singulární řešení

$\int \frac{1}{1+y^2}\,dy \overset{c}{=} \operatorname{arctg} y$ $\qquad \int \frac{-x}{1+x^2}\,dx \overset{c}{=} -\frac{1}{2}\log(1+x^2)$



$\operatorname{arctg} y = -\frac{1}{2}\log(1+x^2) + c$

$\{x \in \mathbb{R};\ -\frac{1}{2}\log(1+x^2) + c \in (-\frac{\pi}{2}, \frac{\pi}{2})\}$

$-\frac{1}{2}\log(1+x^2) + c \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$-\frac{1}{2}\log(1+x^2) \in (-\frac{\pi}{2} - c, \frac{\pi}{2} - c)$

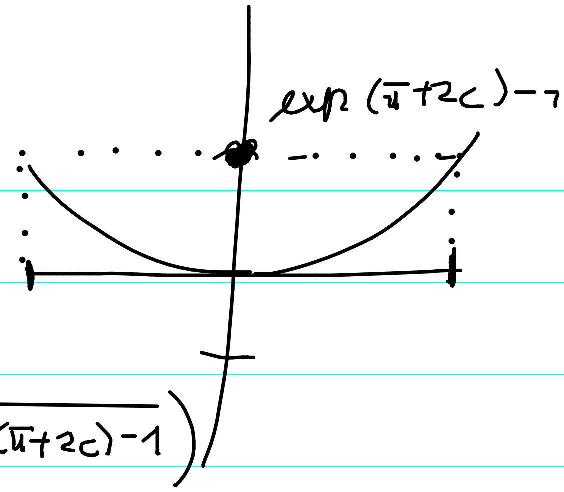
$\log(1+x^2) \in (-\pi + 2c, \pi + 2c)$

$x^2 \in (\exp(-\pi+2c) - 1, \exp(\pi+2c) - 1)$

$\exp(\pi+2c) - 1 \leq 0 \iff c \leq -\frac{\pi}{2}$ $\qquad \emptyset$

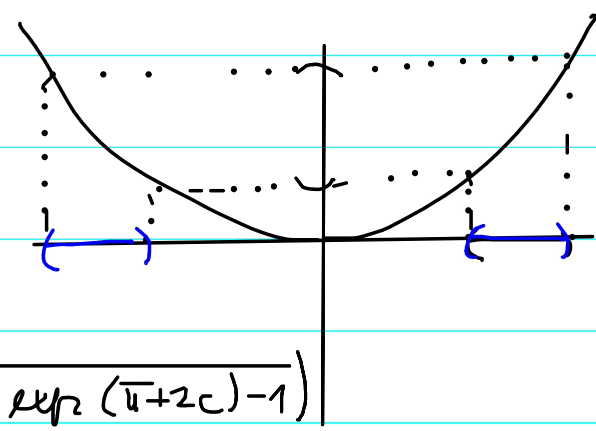
$\exp(-\pi+2c) - 1 < 0 \iff c < \frac{\pi}{2}$

$c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$x \in \left(-\sqrt{\exp(\pi+2c)-1}, \sqrt{\exp(\pi+2c)-1}\right)$

$c \geq \frac{\pi}{2}$



$x \in \left(\sqrt{\exp(-\pi+2c)-1}, \sqrt{\exp(\pi+2c)-1}\right)$

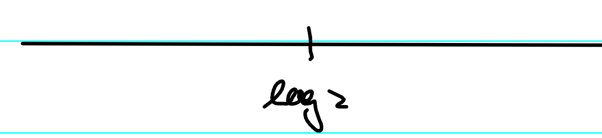
$x \in \left(-\sqrt{\exp(\pi+2c)-1}, -\sqrt{\exp(-\pi+2c)-1}\right)$

$y(x) = \mathrm{tg}\left(-\frac{1}{2}\log(1+x^2)+c\right)$

$y(0) = 1 \implies c = \frac{\pi}{4}$

$y(x) = \mathrm{tg}\left(-\frac{1}{2}\log(1+x^2)+\frac{\pi}{4}\right), \quad x \in \left(-\sqrt{\exp\left(\frac{3}{2}\pi\right)-1}, \sqrt{\exp\left(\frac{3\pi}{2}\right)-1}\right)$

$\boxed{2}$ $y'\underbrace{(2-e^x)} = -3e^x \,\mathrm{tg}\, y \cos^2 y$



$y' = -\frac{3e^x}{2-e^x} \cdot \mathrm{tg}\, y \cdot \cos^2 y$

$$h(x) = \frac{-3e^x}{2-e^x} \qquad g(y) = \operatorname{tg} y \cdot \cos^2 y$$

$\mathcal{D}(g) = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi;\ k \in \mathbb{Z}\}$

$y \in (0, \frac{\pi}{2}) + k\pi \Rightarrow g(y) > 0 \qquad k \in \mathbb{Z}$

$y \in (-\frac{\pi}{2}, 0) + k\pi \Rightarrow g(y) < 0$

$y = k\pi \Rightarrow g(y) = 0$ (1) singulární řešení: $y(x) = k\pi,\ k \in \mathbb{Z}$
$x \in (\log 2, \infty)$

(2) $y \in (0, \frac{\pi}{2}) + k\pi,\ k \in \mathbb{Z}$

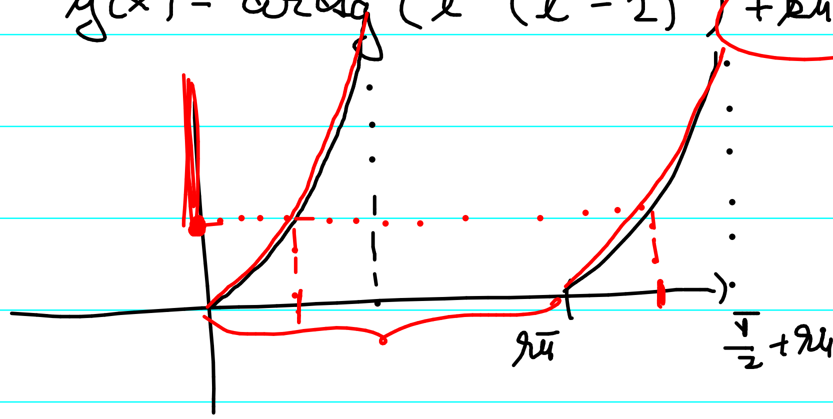
$$\int \frac{1}{\operatorname{tg} y \cos^2 y}\, dy \overset{c}{=} \log|\operatorname{tg} y| = \log(\operatorname{tg} y)$$

$$\int \frac{-3e^x}{2-e^x}\, dx \overset{c}{=} 3\log|2-e^x| = \log(e^x-2)^3$$

$$\underbrace{\log(\operatorname{tg} y)} = \log(e^x-2)^3 + c$$

$$\operatorname{tg} y = \exp(\log(e^x-2)^3 + c) = e^c \cdot (e^x-2)^3$$

$$y(x) = \operatorname{arctg}(e^c(e^x-2)^3) + k\pi$$



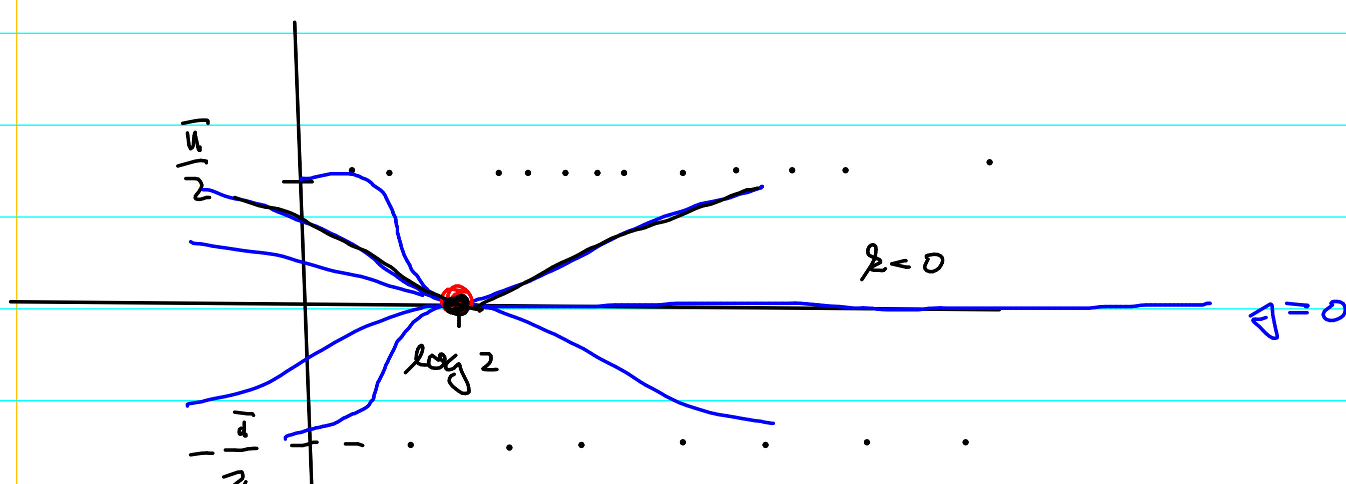
(3) $y \in \left(-\frac{\pi}{2}, 0\right) + k\pi$

$$\log|\operatorname{tg} y| = \log(e^x-2)^3 + c$$

$$\log(-\operatorname{tg} y) = \log(e^x-2)^3 + c$$

$$y(x) = -\operatorname{arctg} e^c (e^x-2)^3 + k\pi, \quad x \in (\log 2, \infty)$$

$$y(x) = \operatorname{arctg} a (e^x-2)^3 + k\pi, \quad x \in (\log 2, \infty)$$



$$\lim_{x \to \log 2^+} \operatorname{arctg} a (e^x-2)^3 + k\pi = k\pi$$

$$y(x) = \begin{cases} \operatorname{arctg} a (e^x-2)^3 + k\pi, & x \in (-\infty, \log 2) \\ k\pi, & x = \log 2 \\ \operatorname{arctg} b (e^x-2)^3 + k\pi, & x = (\log 2, \infty) \end{cases} \qquad \begin{matrix} a, b \in \mathbb{R} \\ k \in \mathbb{Z} \end{matrix}$$

$$y'(2-e^x) = -3e^x \operatorname{tg} y \cdot \cos^2 y$$

$$y(x) = \operatorname{arctg} a(e^x - 2)^3 + \pi 4$$

$$y'(x) = \frac{1}{1 + (a(e^x-2)^3)^2} \cdot a \cdot 3(e^x-2)^2 \cdot e^x$$

$$y'(\log 2) = 0$$