CVIČENÍ Z MA 2, 29.5.2020

$\boxed{1}$ $y' = \frac{1}{x}\sqrt{1-y^2}$

$h(x) = \frac{1}{x}$, $g(y) = \sqrt{1-y^2}$

$\mathcal{D}(h) = (-\infty,0)\cup(0,\infty)$ $\quad$ $\mathcal{D}(g) = [-1,1]$, $g>0$ na $(-1,1)$

$y(x)=1$, $x\in(-\infty,0)$, $x\in(0,\infty)$
$y(x)=-1$, — // —
} singulární řešení

$\int \frac{1}{x}\,dx \stackrel{c}{=} \log|x|$ $\quad x\in(-\infty,0)$, $x\in(0,\infty)$

$\int \frac{1}{\sqrt{1-y^2}}\,dy \stackrel{c}{=} \arcsin y$, $y\in(-1,1)$

$$\arcsin y = \log|x| + c$$

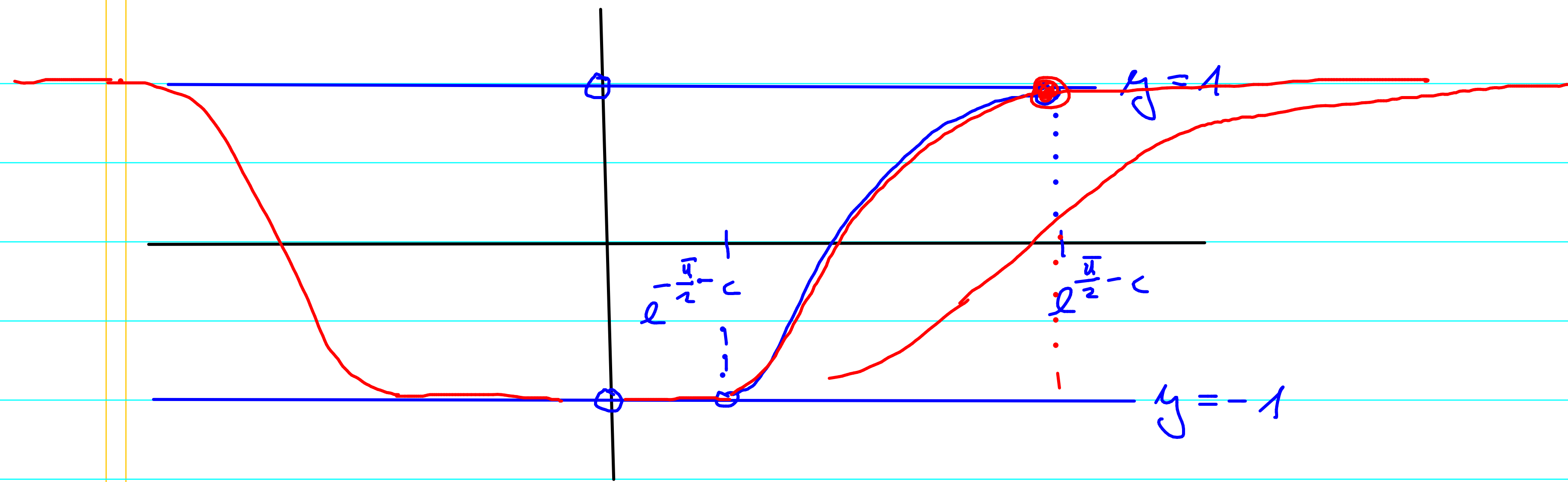
$\log|x| + c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$|x| \in \left(\exp\left(-\frac{\pi}{2}-c\right), \exp\left(\frac{\pi}{2}-c\right)\right)$

$x\in\left(\exp\left(-\frac{\pi}{2}-c\right), \exp\left(\frac{\pi}{2}-c\right)\right)$ $\qquad c\in\mathbb{R}$

$x\in\left(-\exp\left(\frac{\pi}{2}-c\right), \exp\left(-\frac{\pi}{2}-c\right)\right)$

$y(x) = \sin(\log|x| + c)$



$$y(x)=\begin{cases} -1, & x\in\left(0,e^{-\frac{\pi}{2}-c}\right] \\ \sin(\log x+c), & x\in\left(e^{-\frac{\pi}{2}-c},e^{\frac{\pi}{2}-c}\right) \\ 1, & x\in\left[e^{\frac{\pi}{2}-c},\infty\right) \end{cases}$$

$$y'=\underbrace{\frac{1}{x}\sqrt{1-y^2}}_{>0}$$

$y' + p y = q \qquad p, q : (a,b) \to \mathbb{R}$ spojité, $y(x_0) = y_0$

$x_0 \in (a,b)$

$$y(x) = \int_{x_0}^{x} \underbrace{q(t) e^{P(t)}} \, dt \cdot e^{-P(x)} + \underbrace{y_0 e^{-P(x)}}$$

$P' = p$

$y' + p y = 0 \qquad y(x) = k e^{-P(x)}, \quad x \in (a,b) \quad k \in \mathbb{R}$

$y' = -p y \qquad P(x_0) = 0$

$$y(x) = k(x) e^{-P(x)}$$

$$\underbrace{k' e^{-P} + \cancel{k e^{-P} \cdot (-p)}}_{y'} + \cancel{p k e^{-P}} = q$$

$k' e^{-P} = q$

$k' = q e^{P}$

$$k = \int_{x_0}^{x} q(t) e^{P(t)} \, dt + C$$

$\boxed{2}$ $y' - \frac{2}{x} y = \widetilde{2x^3}$, $(0,\infty)$

$p(x) = -\frac{2}{x}$ $\qquad q(x) = 2x^3$ $\qquad P' = p$

$\int \frac{-2}{x} dx \overset{c}{=} -2 \log x$, $x \in (0,\infty)$ $\qquad \underline{P(x) = -2\log x}$

$\int q(t) e^{P(t)} dt = \int 2t^3 \cdot \underbrace{e^{-2\log t}}_{t^{-2}} dt = \int 2t\, dt \overset{c}{=} t^2$

$y(x) = x^2 \cdot e^{-P(x)} + c e^{-P(x)}$

$= x^4 + cx^2$, $x \in (0,\infty)$, $c \in \mathbb{R}$

$\underline{y(1) = 1}$ $\qquad y(1) = 1 + c = 1 \Rightarrow c = 0$

$y(x) = x^4$, $x \in (0,\infty)$

$\boxed{3}$ $y' - \frac{1}{x+1} y = \frac{1}{100}$ $\quad (-1, \infty)$

$p(x) = -\frac{1}{x+1}$

$q(x) = \frac{1}{100}$

$\int p(x)dx = \int -\frac{1}{x+1} dx \overset{c}{=} \underbrace{-\log(x+1)}_{P(x)}, \quad x \in (-1, \infty)$

$\int q(t) e^{P(t)} dt = \int \frac{1}{100} \frac{1}{t+1} dt \overset{c}{=} \frac{1}{100} \log(t+1), \quad t \in (-1, \infty)$

$y(x) = \frac{1}{100} \log(x+1)\, e^{-P(x)} + c\, e^{-P(x)}$

$= \frac{1}{100} \log(x+1) \cdot (x+1) + c(x+1), \quad x \in (-1, \infty)$

$c \in \mathbb{R}$

$\boxed{4}$ $y' + y = x e^{x}$ $\mathbb{R}$

$p(x) = 1$ $P(x) = x$

$q(x) = x e^{x}$

$$\int t e^{t} \cdot e^{t}\,dt = \int \underbrace{t}_{u}\,\underbrace{e^{2t}}_{v'}\,dt = \underbrace{t}_{u}\,\underbrace{\tfrac{1}{2} e^{2t}}_{v} - \int \underbrace{1 \cdot \tfrac{1}{2} e^{2t}}_{u'v}\,dt$$

$$\overset{c}{=} \frac{1}{2} t e^{2t} - \frac{1}{4} e^{2t}$$

$$y(x) = \left(\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}\right) e^{-P} + c e^{-P}$$

$$= \left(\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}\right) e^{-x} + c e^{-x}$$

$$= \frac{1}{2} x e^{x} - \frac{1}{4} e^{x} + c e^{-x}, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}$$