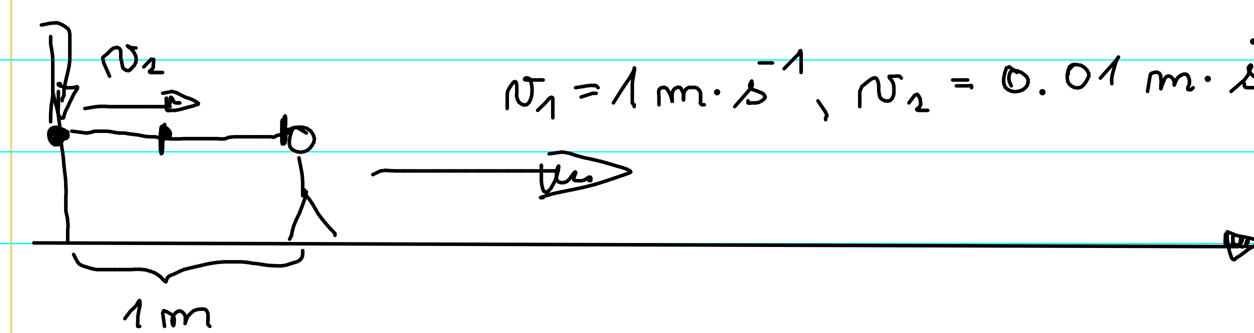
CVIČENÍ Z MATEMATICKÉ ANALÝZY 2, 3.6.2020

$\boxed{1}$ (princezna a mravenec)

$v_1 = 1\, m\cdot s^{-1}$, $v_2 = 0.01\, m \cdot s^{-1}$



$y(t)$... poloha mravence v čase t
$y'(t)$... rychlost mravence v čase t

$$y'(t) = v_2 + \underbrace{\frac{y(t)}{v_1(t+1)}\, v_1} \qquad y(0) = 0$$

$$y' - \frac{1}{t+1}\, y = \frac{1}{100}$$

$$y(t) = \frac{1}{100}\log(t+1)\cdot(t+1) + C(t+1), \quad t \in (-1, \infty)$$

$$y(0) = C = 0$$

$$y(t) = \frac{1}{100}\underbrace{\log(t+1)}\cdot(t+1) > v_1\cdot(t+1) = t+1$$

$$\log(t+1) > 100$$

$$t+1 > e^{100} > \left(2^{10}\right)^{10} > 10^{30}$$

$\boxed{2}$ $m v' = mg - b v$ (volný pád s odporem vzduchu)

$$v' = g - \frac{b}{m} v$$

$$v' + \frac{b}{m} v = g, \quad v(0) = 0 = v_0$$

$$p(t) = \frac{b}{m} \qquad q(t) = g$$

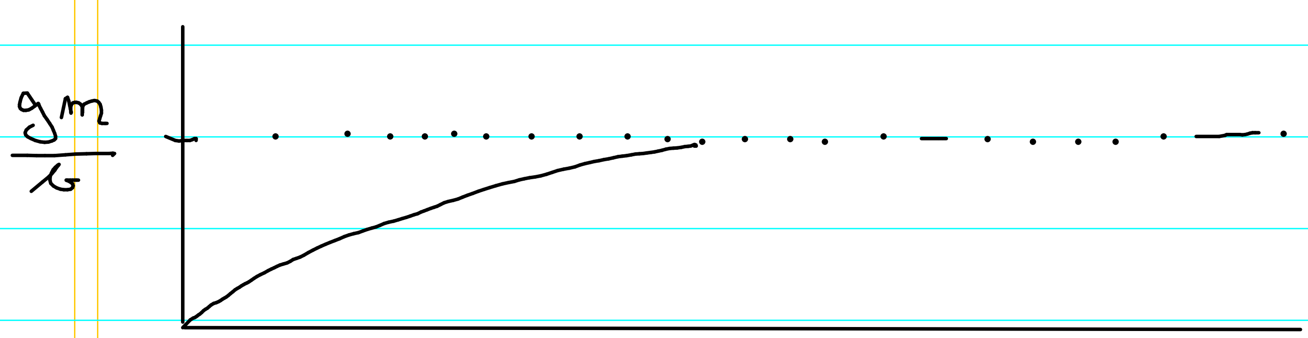
$$P(t) = \frac{b}{m} t$$

$$v(t) = \int_0^t g e^{\frac{b}{m}s} ds \cdot e^{-\frac{b}{m}t} + \underbrace{v_0}_{=0} \cdot e^{-\frac{b}{m}t}$$

$$= \left[g \frac{m}{b} e^{\frac{b}{m}s} \right]_0^t \cdot e^{-\frac{b}{m}t}$$

$$= \frac{gm}{b} \underbrace{e^{\frac{b}{m}t} \cdot e^{-\frac{b}{m}t}} - \frac{gm}{b} \cdot e^{-\frac{b}{m}t}$$

$$= \frac{gm}{b} \left(1 - e^{-\frac{b}{m}t}\right)$$



Lineární diferenciální rovnice n-tého řádu

$$y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = 0$$

$n \in \mathbb{N}$

$a_0, \dots, a_{n-1} \in \mathbb{R}$

$$\lambda^n + a_{n-1} \lambda^{n-1} + \dots + a_1 \lambda + a_0 = 0$$

$\lambda_1, \lambda_2, \dots$ reálné kořeny

$\alpha_1 + \beta_1 i, \dots$ komplexní kořeny

$\boxed{3}$ $y'' + 4y' + 4y = 0$

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda + 2)^2 = 0 \implies \lambda_{1,2} = -2$

F.S. $e^{-2x}, \; x e^{-2x}$

$y(x) = c_1 e^{-2x} + c_2 x e^{-2x}, \; x \in \mathbb{R}, \; c_1, c_2 \in \mathbb{R}$

$\boxed{4}$ $y'' - 3y' + 2y = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$(\lambda - 1)(\lambda - 2) = 0 \qquad \lambda_1 = 1, \; \lambda_2 = 2$

F.S. $e^{1 \cdot x}, \; e^{2 \cdot x}$

$(e^x, \; e^{2x})$

$y(x) = c_1 e^x + c_2 e^{2x}, \; x \in \mathbb{R}, \; c_1, c_2 \in \mathbb{R}$

$\boxed{5}$ $y'' - 6y' + 13y = 0$

$r^2 - 6r + 13 = 0$

$$r_{1,2} = \frac{6 \pm \sqrt{36 - 4 \cdot 13}}{2} = \frac{6 \pm \sqrt{-16}}{2} = 3 \pm 2i$$

F. S. $e^{3x} \cos 2x, e^{3x} \sin 2x$

$$y(x) = c_1 e^{3x} \cos 2x + c_2 e^{3x} \sin 2x, \quad x \in \mathbb{R}, \quad c_1, c_2 \in \mathbb{R}$$

$\boxed{6}$ $y'' + 3y' = 3x e^{-3x}$

$y''(x) + 3y'(x) = 3x e^{-3x}$

$y'' + 3y' = 0$

$r^2 + 3r = 0$

$r(r+3) = 0 \Rightarrow r_1 = 0, r_2 = -3$

F.S. $e^{0 \cdot x}, e^{-3x}$

$1, e^{-3x}$

$y(x) = c_1(x) \cdot 1 + c_2(x) \cdot e^{-3x}$

$y'(x) = \underbrace{c_1'(x) \cdot 1 + c_2'(x) \cdot e^{-3x}}_{=0} + c_1(x) \cdot 0 + c_2(x) \cdot (-3) e^{-3x}$

$y''(x) = c_1'(x) \cdot 0 + c_2'(x)(-3) e^{-3x} + c_1(x) \cdot 0 + c_2(x) \, 9 \, e^{-3x}$

$$y'' + 3y' = c_1' \cdot 0 + c_2'(-3)e^{-3x} + c_1 \cdot 0 + c_2 9 e^{-3x}$$
$$+ 3\left(c_1 \cdot 0 + c_2(-3)e^{-3x}\right)$$
$$= c_1' \cdot 0 + c_2'(-3)e^{-3x} = 3x e^{-3x}$$

$$c_1' \cdot 1 + c_2' e^{-3x} = 0 \qquad c_1'(x),\ c_2'(x)$$
$$c_1' \cdot 0 + c_2'(-3)e^{-3x} = 3x e^{-3x}$$

$$c_2' = -x \qquad c_1' = x e^{-3x}$$

$$\int x e^{-3x}\,dx \overset{c}{=} -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x}$$
$$\int -x\,dx \overset{c}{=} -\frac{1}{2}x^2$$

$$y(x) = \left(-\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x}\right) \cdot 1 + \left(-\frac{1}{2}x^2\right) e^{-3x}$$
$$+ \alpha_1 \cdot 1 + \alpha_2 \cdot e^{-3x},\ x \in \mathbb{R},\ \alpha_1, \alpha_2 \in \mathbb{R}$$