CVIČENÍ Z MATEMATICKÉ ANALÝZY 2

$\boxed{1}$ $y''' - y'' - 2y' = x^3 + 3x^2 + 1$

$\lambda^3 - \lambda^2 - 2\lambda = 0 \qquad 0, 2, -1$

$$x^3 + 3x^2 + 1 = e^{0\cdot x}\Big(\underbrace{(x^3+3x^2+1)}_{P}\cdot \cos(0\cdot x) + \underbrace{0}_{Q}\cdot \sin(0\cdot x)\Big)$$

$\mu = 0, \nu = 0$

$\mu + i\nu = 0$ ... násobnost $= 1$

$\max\{\text{st}\,P, \text{st}\,Q\} = 3$

$y(x) = x^m e^{\mu x}\left(R(x)\cos\nu x + S(x)\sin\nu x\right)$

$\text{st}\,R \leq \max\{\text{st}\,P, \text{st}\,Q\}, \quad \text{st}\,S \leq \max\{\text{st}\,P, \text{st}\,Q\}$

$y(x) = x\left(ax^3 + bx^2 + cx + d\right) = ax^4 + bx^3 + cx^2 + dx$

$y'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$y''(x) = 12ax^2 + 6bx + 2c$

$y'''(x) = 24ax + 6b$

$x^0: \quad 6b - 2c - 2d = 1 \qquad x^2: \ -12a - 6b = 3 \qquad b = \frac{1}{6}(-12a - 3) = -2a - \frac{1}{2}$

$x^1: \quad 24a - 6b - 4c = 0 \qquad x^3: \ -8a = 1 \implies a = -\frac{1}{8} \qquad = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$c = \frac{1}{4}(24a - 6b) = 6a - \frac{3}{2}b = -\frac{6}{8} + \frac{3}{8} = -\frac{3}{8}$

$d = \frac{1}{2}(6b - 2c - 1) = 3b - c - \frac{1}{2} = -\frac{3}{4} + \frac{3}{8} - \frac{4}{8} = \frac{1}{8}(-6 + 3 - 4) = -\frac{7}{8}$

$$y(x) = -\frac{1}{8}x^4 - \frac{1}{4}x^3 - \frac{3}{8}x^2 - \frac{7}{8}x + \alpha_1 + \alpha_2 e^{2x} + \alpha_3 e^{-x}, \quad x \in \mathbb{R} \qquad \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$$

$\boxed{2}$ $\underbrace{y''' - y'' - 2y'}_{L(y)} = \underbrace{e^{2x}}_{f_1} + \underbrace{x^3 + 3x^2 + 1}_{f_2}$

$L(y_1) = f_1$

$L(y_2) = f_2$ $\qquad\qquad 1, e^{-x}, e^{2x}$

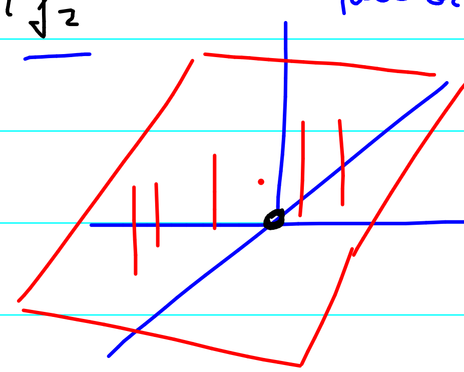
$y(x) = \frac{1}{6} x e^{2x} - \frac{1}{8} x^4 - \frac{1}{4} x^3 - \frac{3}{8} x^2 - \frac{7}{8} x + \alpha_1 + \alpha_2 e^{2x} + \alpha_3 e^{-x}$, $x \in \mathbb{R}$, $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$

linearita

$L(y_1 + y_2) = L(y_1) + L(y_2) = f_1 + f_2$

$(C^\infty(\mathbb{R}), +, \cdot)$

vektorový prostor



$L(y_1) = f_1$

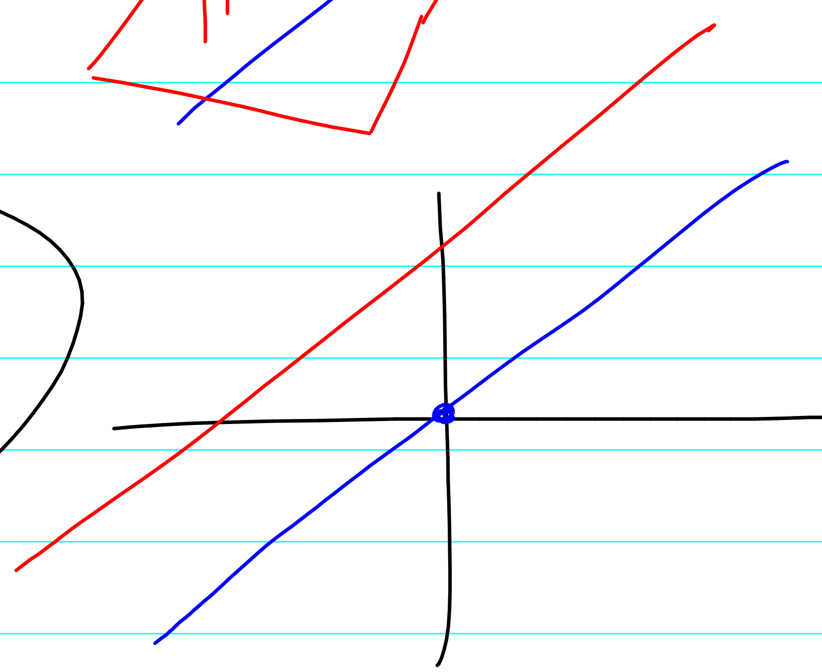
$L(y_2) = f_2$

$L(y) = f_1 + f_2 \quad (*)$

$L(y) = 0$

$L(y) = f$ $\qquad L(y_p) = f$

$y_p + y_h$



$\boxed{3}$ $y''' + 3y'' + 3y' + y = x\cos x + \sin x$

$r^3 + 3r^2 + 3r + 1 = 0$

$$(r+1)^3 = 0 \implies r_{1,2,3} = -1$$

F.S. $e^{-x}, \; xe^{-x}, \; x^2 e^{-x}$

$\underset{}{e^{0\cdot x}}(\underset{P}{x}\cdot \cos x + \underset{Q}{1}\cdot \sin x)$ $\quad \mu = 0, \; \nu = 1 \quad$ $0 + 1\cdot i = i$ ... není kořenem

$\implies m = 0$

$\max\{\text{st } P, \text{st } Q\} = 1$

$y(x) = (ax+b)\cdot \cos x + (cx+d)\sin x$

$y'(x) = a\cdot \cos x - (ax+b)\sin x + c\sin x + (cx+d)\cos x$

$\quad = (cx + a + d)\cos x + (-ax - b + c)\sin x$

$y''(x) = c\cdot \cos x - (cx+a+d)\sin x - a\sin x + (-ax-b+c)\cos x$

$\quad = (-ax - b + 2c)\cos x + (-cx - d - 2a)\sin x$

$y'''(x) = -a\cos x - (-ax-b+2c)\sin x - c\sin x + (-cx-d-2a)\cos x$

$\quad = (-cx - d - 3a)\cos x + (ax + b - 3c)\sin x$

$\sin x$: $b - 3c + 3(-d - 2a) + 3(-b + c) + d = 1$

$\cos x$: $-d - 3a + 3(-b + 2c) + 3(a + d) + b = 0$

$x\sin x$: $a - 3c - 3a + c = 0 \qquad -2a - 2c = 0$

$x\cos x$: $-c - 3a + 3c + a = 1 \qquad -2a + 2c = 1$

$$\left.\begin{aligned} -2a - 2c &= 0 \\ -2a + 2c &= 1 \end{aligned}\right\} -4a = 1 \implies a = -\frac{1}{4}$$

$$c = \frac{1}{4}$$

$$-6a - 2b - 2d = 1 \qquad -2b - 2d = 1 + 6a = 1 - \frac{6}{4} = -\frac{1}{2}$$

$$-2b + 6c + 2d = 0 \qquad -2b + 2d = -6c = -\frac{3}{2}$$

$$-4b = -2 \Rightarrow b = \frac{1}{2} \qquad 2d = -\frac{3}{2} + 2b = -\frac{3}{2} + 1 = -\frac{1}{2}$$

$$d = -\frac{1}{4}$$

$$y(x) = \left(-\frac{1}{4}x + \frac{1}{2}\right)\cos x + \left(\frac{1}{4}x - \frac{1}{4}\right)\sin x + \alpha_1 e^{-x} + \alpha_2 x e^{-x} + \alpha_3 x^2 e^{-x}, \quad x \in \mathbb{R}$$

$$\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$$

$\boxed{4}$ $y'' - 2y' + 2y = e^x \cos x + e^x \sin x$

$r^2 - 2r + 2 = 0$

$r_{1,2} = \frac{1}{2}\left(2 \pm \sqrt{4 - 8}\right) = \frac{1}{2}(2 \pm 2i) = 1 \pm i$

F.S. $e^x \cos x, e^x \sin x$

$e^x(\underset{P}{1} \cdot \cos x + \underset{Q}{1} \cdot \sin x)$ $\qquad \mu + i\nu = 1 + i$ … násobnosť 1

$\max\{st P, st Q\} = 0$

$y(x) = x \cdot e^x (a \cos x + b \sin x)$

$(uvw)' = u'vw + uv'w + uvw'$

$$y(x) = \frac{1}{2} x e^x (\sin x - \cos x) + \alpha_1 e^x \sin x + \alpha_2 e^x \cos x, \quad x \in \mathbb{R}, \quad \alpha_1, \alpha_2 \in \mathbb{R}$$