CVIČENÍ Z MATEMATICKÉ ANALÝZY 2, 12.6.2020

$\boxed{1}$ $f(x) = x\sqrt[3]{\underbrace{1-3x}} - x\sqrt[4]{\underbrace{1-4x}}$, $x \in (-\infty, \frac{1}{4})$

$T_4^{f,0}$

$$(1+x)^\alpha = 1 + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \binom{\alpha}{3}x^3 + o(x^3)$$

$$= 1 + \alpha x + \frac{1}{2}\alpha(\alpha-1)x^2 + \frac{1}{6}\alpha(\alpha-1)(\alpha-2)x^3 + o(x^3)$$

$$(1-3x)^{\frac{1}{3}} = 1 + \frac{1}{3}(-3x) + \frac{1}{2}\frac{1}{3}\frac{-2}{3}9x^2 + \frac{1}{6}\frac{1}{3}\frac{-2}{3}\frac{-5}{3}(-3x)^3 + o(x^3)$$

$$= \underline{1} - \underline{x} - \underline{x^2} - \underline{\frac{5}{3}x^3} + o(x^3)$$

$$(1-4x)^{\frac{1}{4}} = 1 + \frac{1}{4}(-4x) + \frac{1}{2}\frac{1}{4}\frac{-3}{4}(-4x)^2 + \frac{1}{6}\frac{1}{4}\cdot\frac{-3}{4}\cdot\frac{-7}{4}(-4x)^3 + o(x^3)$$

$$= \underline{1} - \underline{x} - \underline{\frac{3}{2}x^2} - \underline{\frac{7}{2}x^3} + o(x^3)$$

$$f(x) = x\left(\frac{1}{2}x^2 + \frac{21-10}{6}x^3 + o(x^3)\right) = \frac{1}{2}x^3 + \frac{11}{6}x^4 + o(x^4)$$

$$T_4^{f,0}(x) = \frac{1}{2}x^3 + \frac{11}{6}x^4$$

$$a\sin\left(\frac{x}{a}\right) = a\left(\frac{x}{a} - \frac{1}{6}\frac{x^3}{a^3} + o(x^4)\right) = x - \frac{1}{6}\frac{1}{a^2}x^3 + o(x^4)$$

$$\sin x = x - \frac{1}{6}x^3 + o(x^4)$$

$$a\sin\left(\frac{x}{a}\right) - \sin x = \frac{1}{6}\left(1 - \frac{1}{a^2}\right)x^3 + o(x^4)$$

$$\lim_{x\to 0}\frac{f(x)}{a\sin(\frac{x}{a}) - \sin x} = \lim_{x\to 0}\frac{\frac{1}{2}x^3 + \frac{11}{6}x^4 + o(x^4)}{\frac{1}{6}\left(1-\frac{1}{a^2}\right)x^3 + o(x^4)} = \frac{3}{1-\frac{1}{a^2}} = 6$$

$$1 - \frac{1}{a^2} = \frac{1}{2} \Rightarrow a^2 = 2 \underset{a>0}{\Rightarrow} \underline{a = \sqrt{2}}$$

$$\boxed{2} \quad \underbrace{\sum_{n=1}^{\infty} \frac{\sqrt[3]{n+2} - \sqrt[3]{n+1}}{n^{\alpha}}}_{a_n > 0}$$

$$\sqrt[3]{n+2} - \sqrt[3]{n+1} = \frac{1}{(n+2)^{\frac{2}{3}} + (n+2)^{\frac{1}{3}}(n+1)^{\frac{1}{3}} + (n+1)^{\frac{2}{3}}}$$

$$= \frac{1}{n^{2/3}} \cdot \frac{1}{\left(1+\frac{2}{n}\right)^{2/3} + \left(1+\frac{2}{n}\right)^{\frac{1}{3}}\left(1+\frac{1}{n}\right)^{\frac{1}{3}} + \left(1+\frac{1}{n}\right)^{2/3}}$$

$$b_n = \frac{1}{n^{\alpha}} \cdot \frac{1}{n^{2/3}} = \frac{1}{n^{\alpha+2/3}}$$

$$\lim \frac{a_n}{b_n} = \frac{1}{3} \in (0, \infty)$$

$$\sum a_n \text{ K} \Leftrightarrow \sum b_n \text{ K} \Leftrightarrow \alpha + \frac{2}{3} > 1$$

$$\Leftrightarrow \alpha > \frac{1}{3}$$

$\boxed{3}$ $y'' - y' - 2y = x^2 e^{2x}$

$\lambda^2 - \lambda - 2 = 0$

$(\lambda+1)(\lambda-2) = 0 \qquad \lambda_1 = -1,\ \lambda_2 = 2$

F.S. e^{-x}, e^{2x}

$x^2 e^{2x} = e^{2x}(\underbrace{x^2}_{P} \cdot \cos 0 \cdot x + \underbrace{0}_{Q} \cdot \sin 0 \cdot x)$

$\mu = 2, \nu = 0 \qquad \mu + i\nu = 2$ ... násobnost 1

$\max\{\text{st}\,P, \text{st}\,Q\} = 2$

$y(x) = x e^{2x}(ax^2 + bx + c) = e^{2x}(ax^3 + bx^2 + cx)$

$y'(x) = e^{2x}(3ax^2 + 2bx + c) + 2e^{2x}(ax^3 + bx^2 + cx)$

$= e^{2x}(2ax^3 + (2b+3a)x^2 + (2c+2b)x + c)$

$y''(x) = e^{2x}(6ax^2 + 2(2b+3a)x + 2c + 2b) + 2e^{2x}(2ax^3 + (2b+3a)x^2$

$+ (2c+2b)x + c)$

$= e^{2x}(4ax^3 + (12a + 4b)x^2 + (8b + 6a + 4c)x + 2b + 4c)$

$x^3 e^{2x}$: $4a - 2a - 2a = 0$

$x^2 e^{2x}$: $12a + 4b - 2b - 3a - 2b = 1 \Rightarrow 9a = 1 \Rightarrow a = \frac{1}{9}$

$x e^{2x}$: $8b + 6a + 4c - 2c - 2b - 2c = 0 \Rightarrow 6a + 6b = 0 \Rightarrow b = -\frac{1}{9}$

e^{2x}: $2b + 4c - c = 0 \Rightarrow 2b + 3c = 0 \Rightarrow c = \frac{2}{27}$

$y(x) = \frac{1}{9}x^3 e^{2x} - \frac{1}{9}x^2 e^{2x} + \frac{2}{27}e^{2x} + \alpha_1 e^{-x} + \alpha_2 e^{2x},\ x \in \mathbb{R} \qquad \alpha_1, \alpha_2 \in \mathbb{R}$